Non-Supersymmetric Strings and Finiteness

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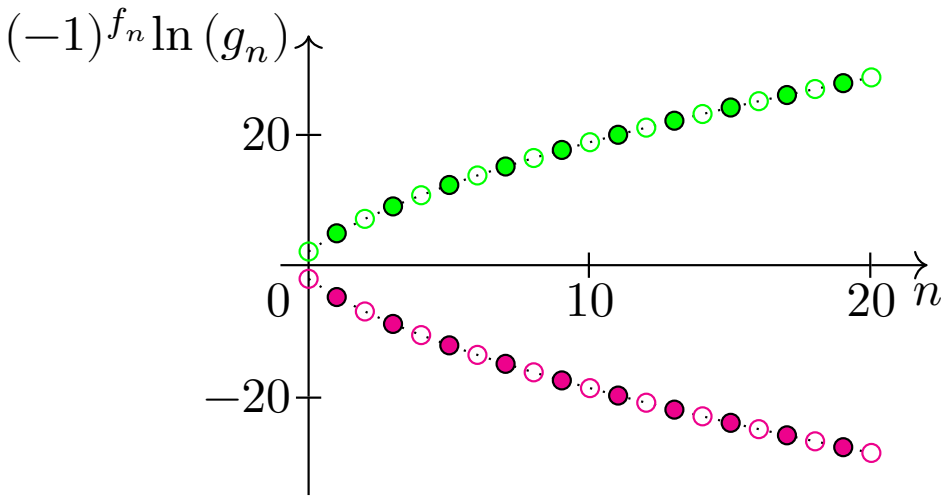
based on collaborations with Niccolò Cribiori, Susha Parameswaran and Timm Wrase

1. JHEP 04 (2021) 099 [hep-th/2012.04677]
2. JHEP 01 (2022) 127 [hep-th/2110.11973]

example: Dp-brane or anti-Dp-brane on Op-plane ($m_n^2 = n/\alpha'$)

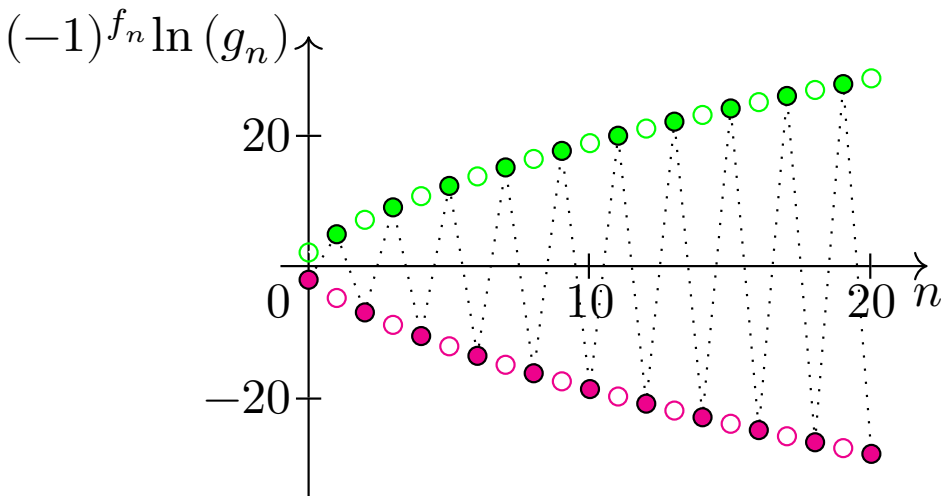
review: Angelantonj, Sagnotti - Phys. Rept. 371 (2002) 1 [hep-th/0204089]



- bosons
- fermions

Dp-brane on Op-plane:

$\Lambda_{p+1} = 0$, i.e. SUSY



anti-Dp-brane on Op-plane:

non-SUSY,

but $|\Lambda_{p+1}| < \infty$ $(p < 7)$

observation:

- non-SUSY strings with:
  - non-zero net degeneracies $(-1)^{f_n} g_n \gtrless 0$
  - possibly finite loop corrections e.g. $\Lambda_D < \infty$

goal:

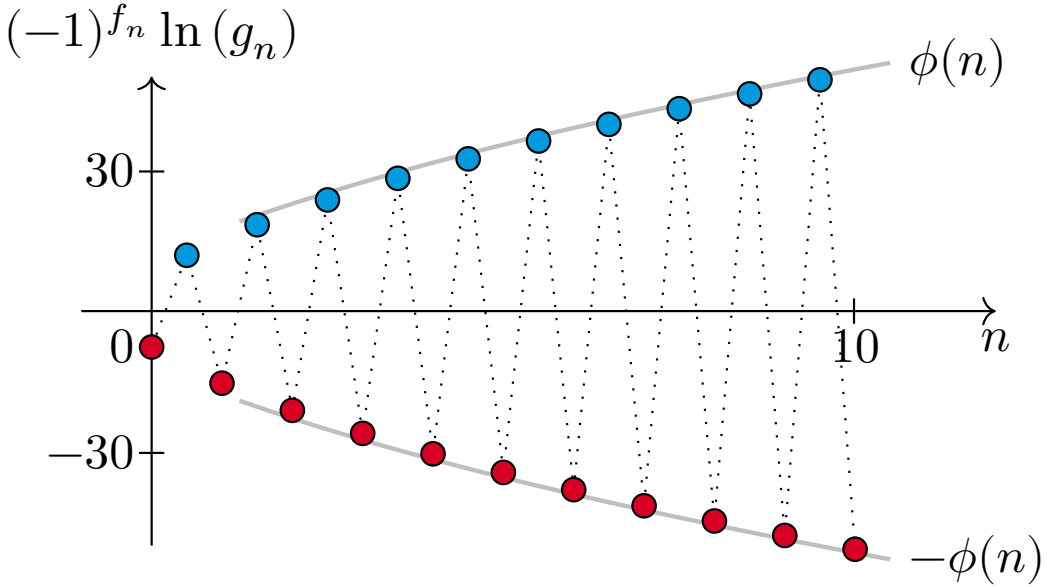
- interpret string-theory finiteness as due to boson-fermion cancellations
  entailing the whole string spectrum, i.e. with no cutoffs,
  even in the absence of SUSY (*misaligned supersymmetry*)

Dienes - Nucl. Phys. B429 (1994) 533 [hep-th/9402006]
Dienes - Nucl. Phys. B611 (2001) 146 [hep-th/0104274]

Closed-String Misaligned Supersymmetry

example: heterotic $\mathrm{SO}(16)\times\mathrm{SO}(16)$-theory ($M_n^2 = 4n/\alpha'$)

Álvarez-Gaumé, Ginsparg, Moore, Vafa - Phys. Lett. B171 (1986) 155



$g_n > 0$

$\Lambda_{10} < \infty$

- intuition: $\Lambda_{10} < \infty$ because $\phi_b(n) - \phi_f(n) = 0$

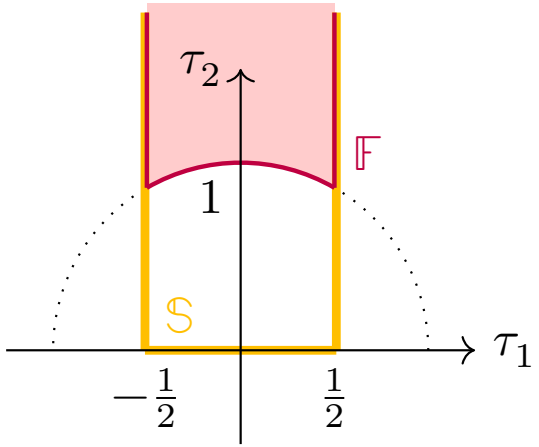
note:

- only closed strings
- only leading order in HRR-sum

Closed-String Modular Invariance and Finiteness

partition function $Z = Z(\tau, \bar{\tau})$,
modular invariance (+ no tachyons):

$$\tilde{\Lambda}_D = -\frac{1}{2}\left[\frac{1}{2\pi\sqrt{\alpha'}}\right]^D \int_{\mathbb{F}} \frac{\mathrm{d}^2\tau}{\tau_2^2}\, Z(\tau,\bar{\tau}) < \infty$$



- Kutasov-Seiberg identity:

$$\tilde{\Lambda}_D = -\frac{\pi}{6}\left[\frac{1}{2\pi\sqrt{\alpha'}}\right]^D \lim_{\tau_2\to 0^+}\left[\tau_2^{1-D/2}\sum_{n\in\frac{1}{2}\mathbb{N}_0}(-1)^{f_n} g_n\, \mathrm{e}^{-4\pi\tau_2 n}\right] < \infty$$

Kutasov, Seiberg - Nucl. Phys. B358 (1991) 600
Angelantonj, Cardella, Elitzur, Rabinovici - JHEP 02 (2011) 024 [hep-th/1012.5091]

interpretation:

finiteness due to boson-fermion cancellations
entailing the whole tower of physical states, i.e. misaligned SUSY

non-SUSY open-string theories: similar structure, but no modular invariance

- partition function: $M(\mathrm{i}t) = \sum_{n\in\mathbb{N}_0} (-1)^{f_n} g_n \, \mathrm{e}^{-2\pi t n}$
- one-loop cosmological constant: $\Lambda_{p+1} = -\frac{1}{(2\pi\sqrt{\alpha'})^{p+1}} \int_0^\infty \frac{\mathrm{d}t}{2t} \frac{M(\mathrm{i}t)}{(2t)^{\frac{p+1}{2}}}$
  - possible divergence for $t \sim 0^+$ (if no tachyons, region $t \sim \infty$ is fine)

example: anti-Dp-brane on Op-plane

$$M(\tau) = -\frac{1}{2} \frac{V_8 + S_8}{\eta^8} \left[\tau + \frac{1}{2}\right] = -\frac{8\,\eta^{16}(\tau)\,\eta^{16}(4\tau)}{\eta^{40}(2\tau)}$$

Hardy-Ramanujan-Rademacher Expansion for η-quotients

Kani, Vafa - Commun. Math. Phys. 130 (1990) 529
Sussman '17, math-ph/1710.03415

observation ($q(\tau) = \mathrm{e}^{2\pi \mathrm{i} \tau}$, $\delta_m \in \mathbb{Z}$):

- η-quotient: $M(\tau) = \prod_{m=1}^{\infty} [\eta(m\tau)]^{\delta_m} = \sum_{n=0}^{\infty} a_n q(\tau)^n$
- coefficients as HRR-expansions ($n > 0$):

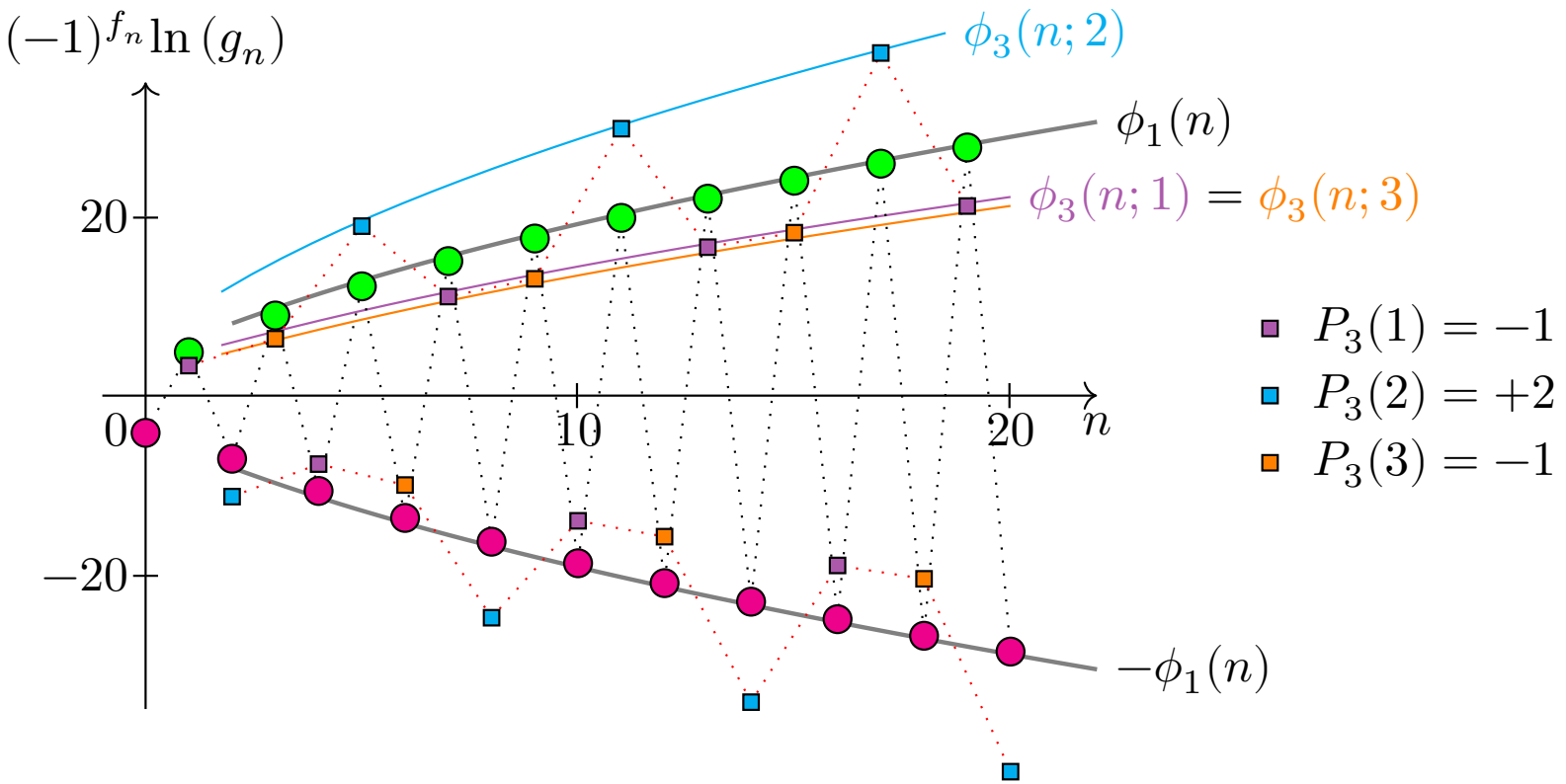
$$a_n = (-1)^{f_n} g_n = \sum_{\alpha \in A \subseteq \mathbb{N}} P_\alpha(n) f_\alpha(n)$$

  - $f_\alpha(n)$: simple function, real even for $n \in \mathbb{R}_0^+$
  - $P_\alpha(n)$: complicated sum of phases, but
    - $\alpha = 1$: $P_1(n) = 1$
    - $\alpha > 1$: periodicity $P_\alpha(n) = P_\alpha(n + \alpha)$, with $\sum_{\beta=1}^{\alpha} P_\alpha(\beta) = 0$

General Envelope Functions

example: anti-Dp-brane on Op-plane

- $\alpha = 1$ (leading order): $\pm\phi_1(n) = \pm f_1(n)$
- $\alpha = 3$ (next-to-leading order): $\pm\phi_3(n;\beta) = \pm P_3(\beta) f_3(n)$



- cancellation of exponential divergences!

series rearrangement:

$$M(\mathrm{i}t) = (-1)^{f_0} g_0 + \sum_{\alpha\in A} \sum_{\beta=1}^{\alpha} P_\alpha(\beta)\left[g_\alpha^+(t;\beta) - g_\alpha^-(t;\beta)\right]$$

- series computation: $g_\alpha^\pm(t;\beta) \overset{t\sim 0^+}{\simeq} A_\alpha\, t^{-B_\alpha}\, \mathrm{e}^{\frac{C_\alpha}{t}} + \sum_{l=0}^{\infty} b_l^\pm(\alpha;\beta) t^l$
- absence of exponential divergences as $t \sim 0^+$:
  - $\alpha = 1$: $g_1^+(t;0) - g_1^-(t;0) \overset{t\sim 0^+}{\simeq} \sum_{l=0}^{\infty} b_l(1;0)\, t^l$
  - $\alpha > 1$: $(\beta,\pm)$-independent divergences, hence cancellation

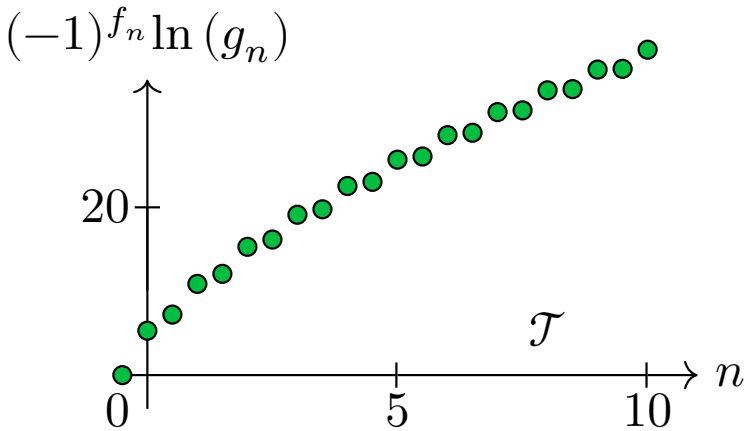
Non-Supersymmetric Closed Strings

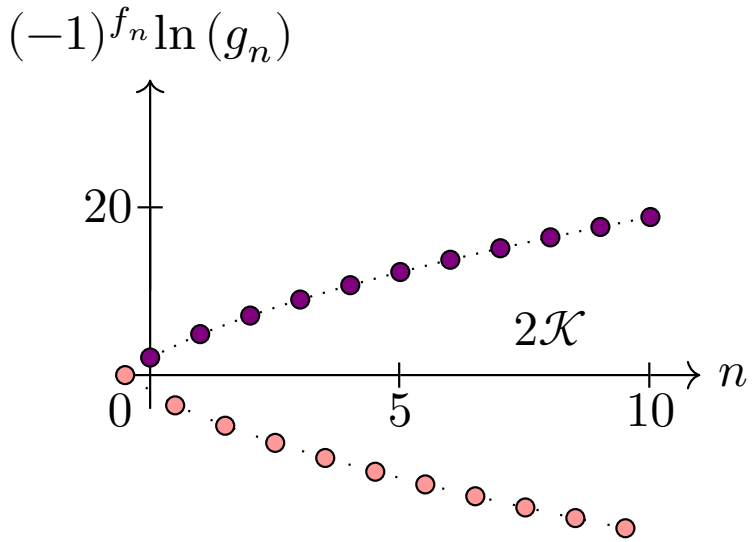
methods for $\mathcal{A}$, $\mathcal{M}$ can be adapted for non-SUSY closed-string sectors ($\mathcal{T}$, $\mathcal{K}$)

- heterotic $\mathrm{SO}(16)\times\mathrm{SO}(16)$-theory
- closed-string sector of type 0'B theory

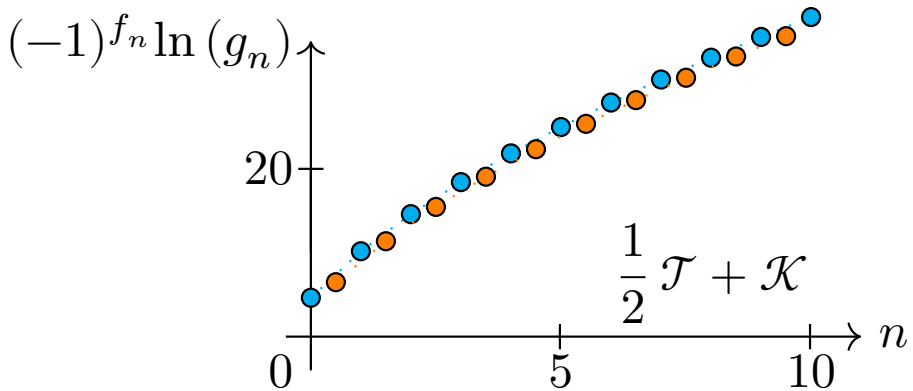
Sagnotti, SUSY 95 [hep-th/9509080]

Sagnotti - Nucl. Phys. Proc. Suppl. 56B (1997) 332 [hep-th/9702093]





total amplitude:



Recap

- developments in misaligned SUSY:
  - all-order cancellations in the HRR-sums, both open and closed strings
  - mathematical description of the exponential cancellations in the one-loop cosmological constant

  examples: Sugimoto model, het-$\mathrm{SO}(16)\times\mathrm{SO}(16)$, type 0'B theory

- new directions:
  - more realistic constructions: e.g. multiple branes, compactifications, higher-loop corrections
  - different observables: e.g. masses

    Abel, Dienes - Phys. Rev. D 104 (2021) 12, 126032 [hep-th/2106.04622]
  - note: non-SUSY strings – e.g. anti-D-branes – with SUSY-like tools (another example: constrained superfields for non-linear SUGRA)

    Cribiori, Roupec, Tournoy, Van Proeyen, Wrase - JHEP 07 (2020) 189 [hep-th/2004.13110]

Thank you!